

A Survey of Probability Concepts

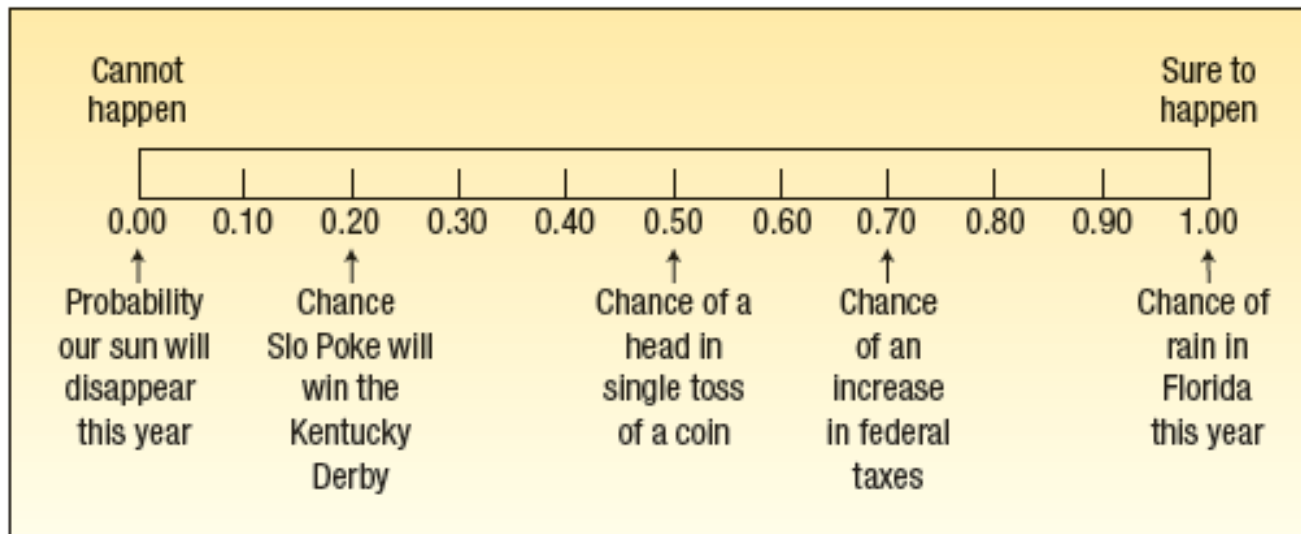
Chapter 5

Learning Objectives

- LO5-1 Define the terms probability, experiment, event, and outcome
- LO5-2 Assign probabilities using a classical, empirical, or subjective approach
- LO5-3 Calculate probabilities using the rules of addition
- LO5-4 Calculate probabilities using the rules of multiplication
- LO5-5 Compute probabilities using a contingency table
- LO5-6 Calculate probabilities using Bayes' theorem
- LO5-7 Determine the number of outcomes using principles of counting

Probability

PROBABILITY A value between 0 and 1 inclusive that represents the likelihood a particular event happens.



Probability

EXPERIMENT A process that leads to the occurrence of one and only one of several possible results.

OUTCOME A particular result of an experiment.

EVENT A collection of one or more outcomes of an experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None is over 60 One is over 60 Two are over 60 ... 29 are over 60 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

Classical Probability

- ▶ The classical definition of probability applies when there are n equally likely outcomes to an experiment

CLASSICAL PROBABILITY

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

[5-1]

MUTUALLY EXCLUSIVE The occurrence of one event means that none of the other events can occur at the same time.

COLLECTIVELY EXHAUSTIVE At least one of the events must occur when an experiment is conducted.

a one-spot		a four-spot	
a two-spot		a five-spot	
a three-spot		a six-spot	

Empirical Probability

- ▶ The empirical definition occurs when the number of times an event happens is divided by the number of outcomes

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

LAW OF LARGE NUMBERS Over a large number of trials, the empirical probability of an event will approach its true probability.

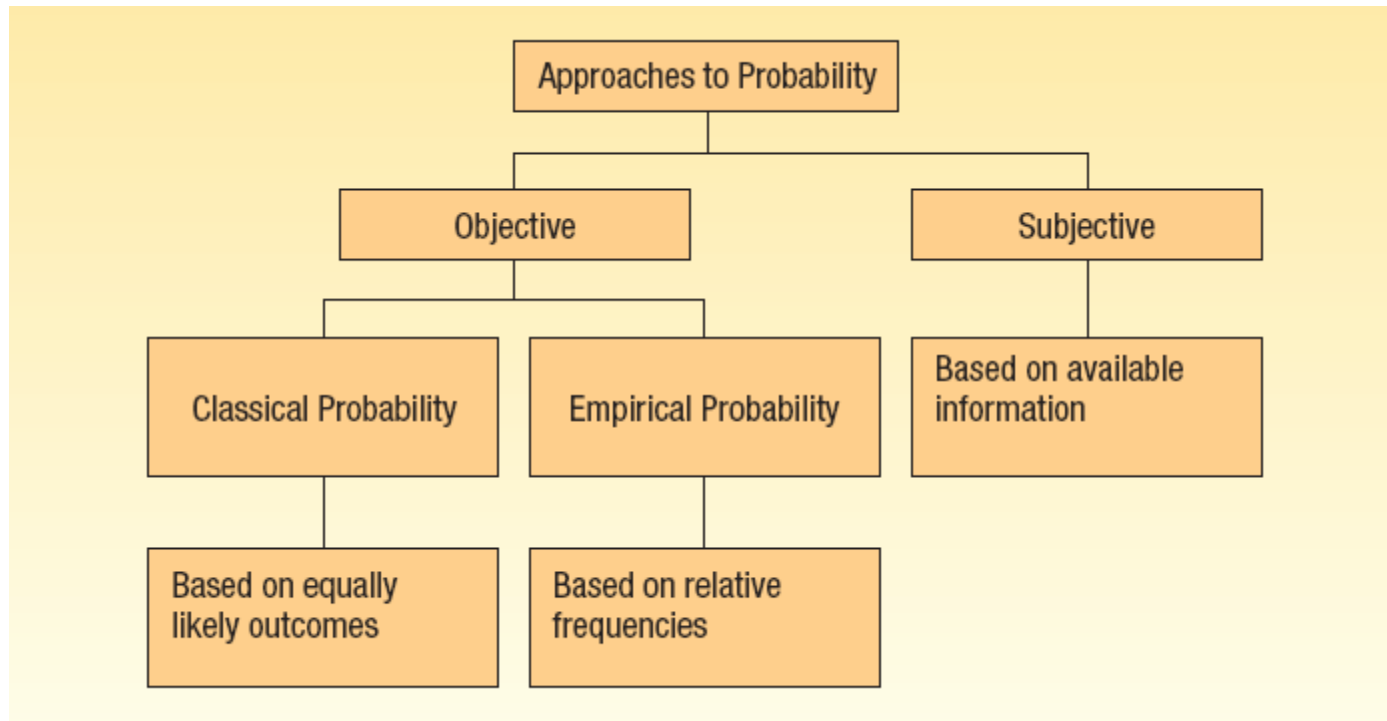
$$\begin{aligned} \text{Probability of a successful flight} &= \frac{\text{Number of successful flights}}{\text{Total number of flights}} \\ &= \frac{121}{123} = 0.98 \end{aligned}$$

Subjective Probability

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

- ▶ Examples of subjective probability are
- ▶ Estimating the likelihood the New England Patriots will be in the Super Bowl next year
- ▶ Estimating the likelihood the U.S. budget deficit will be reduced by half in the next 10 years

Summary of Approaches to Probability



Exercise (1 / 2)

Exercise 2: A quality control inspector selects a part to be tested. The part is then declared acceptable, repairable, or scrapped. Then another part is tested. List the possible outcomes of this experiment regarding two parts.

Exercise 3: A survey of 34 students at the Wall College of Business showed the following majors: Suppose you select a student and observe his or her major.

- What is the probability he or she is a management major?
- Which concept of probability did you use to make this estimate?

Accounting	10
Finance	5
Economics	3
Management	6
Marketing	10

Exercise (2/2)

Exercise 6: A firm will promote two employees out of a group of six men and three women.

- List the chances of this experiment if there is particular concern about gender equity.
- Which concept of probability would you use to estimate these probabilities?

Exercise 8: A sample of 2,000 licensed drivers revealed the following number of speeding violations.

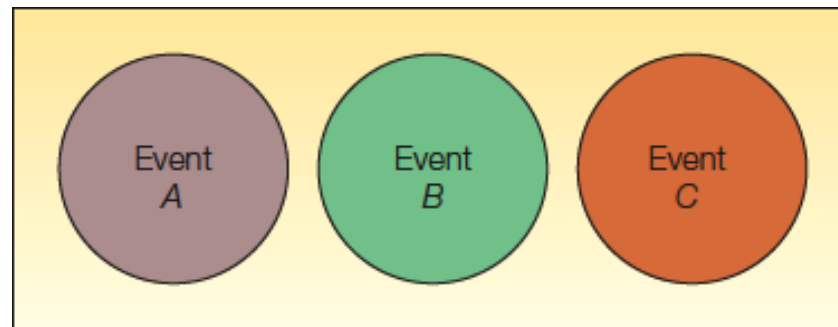
- What is the experiment?
- List one possible event.
- What is the probability that a particular driver had exactly two speeding violations?
- What concept of probability does this illustrate?

Number of Violations	Number of Drivers
0	1,910
1	46
2	18
3	12
4	9
5 or more	5
Total	2,000

Rules of Addition

- ▶ The rules of addition refer to the probability that any two or more events can occur
- ▶ The special rule of addition is used when the events are mutually exclusive

SPECIAL RULE OF ADDITION $P(A \text{ or } B) = P(A) + P(B)$



Rules of Addition Example

A machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4,000 packages filled in the past month revealed:

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	<i>A</i>	100	.025
Satisfactory	<i>B</i>	3,600	.900
Overweight	<i>C</i>	300	.075
		4,000	1.000

← $\frac{100}{4,000}$

- ▶ What is the probability that a particular package will be either underweight or overweight?

$$P(A \text{ or } C) = P(A) + P(C) = .025 + .075 = .10$$

Complement Rule

- ▶ The complement rule is used to determine the probability of an event happening by subtracting the probability of an event not happening

COMPLEMENT RULE $P(A) = 1 - P(\sim A)$

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	<i>A</i>	100	.025
Satisfactory	<i>B</i>	3,600	.900
Overweight	<i>C</i>	300	.075
		4,000	1.000

← $\frac{100}{4,000}$

- ▶ You can also use the complement rule

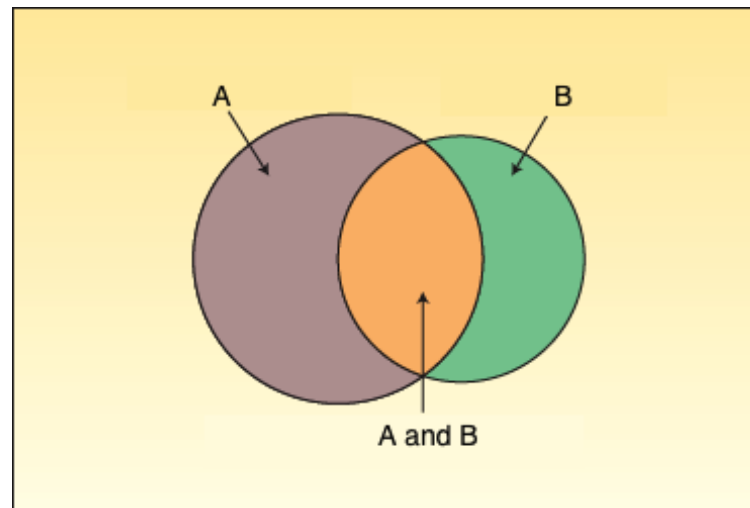
$$P(A \text{ or } C) = P(\sim B) = 1 - P(B) = 1 - .900 = .10$$

General Rule of Addition

- ▶ The general rule of addition is used when the events are not mutually exclusive

GENERAL RULE OF ADDITION $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.



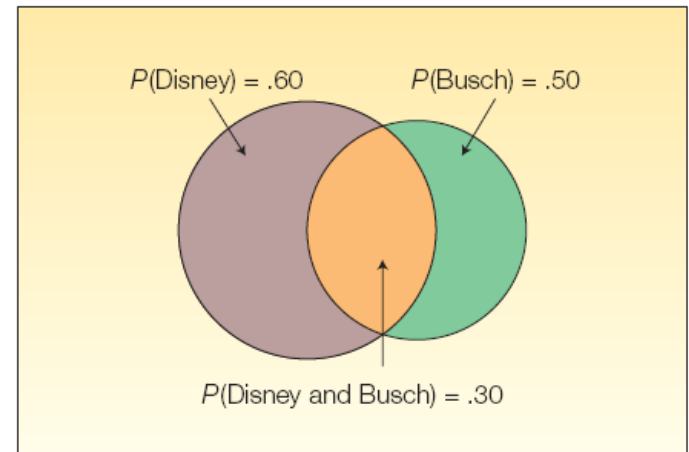
General Rule of Addition Example

A sample of 200 tourists in Florida shows 120 went to Disney, 100 went to Busch Gardens, and 60 visited both.

$$P(\text{Disney}) = 120/200 = .60$$

$$P(\text{Busch}) = 100/200 = .50$$

$$P(\text{Disney and Busch}) = 60/200 = .30$$



$$\begin{aligned} P(\text{Disney or Busch}) &= P(\text{Disney}) + P(\text{Busch}) - P(\text{Disney and Busch}) \\ &= .60 + .50 - .30 = .80 \end{aligned}$$

Exercise (1 / 2)

Exercise 12: The events X and Y are mutually exclusive. Suppose $P(X) = .05$ and $P(Y) = .02$. What is the probability of either X or Y occurring? What is the probability that neither X nor Y will happen?

Exercise 13: A study of 200 advertising firms revealed their income after taxes:

- What is the probability an advertising firm selected at random has under \$1 million in income after taxes?
- What is the probability an advertising firm selected at random has either an income between \$1 million and \$20 million, or an income of \$20 million or more? What rule of probability was applied?

Income after Taxes	Number of Firms
Under \$1 million	102
\$1 million to \$20 million	61
\$20 million or more	37

Exercise (2/2)

Exercise 16: Two coins are tossed. If A is the event “two heads” and B is the event “two tails,” are A and B mutually exclusive? Are they complements?

Exercise 20. A student is taking two courses, history and math. The probability the student will pass the history course is .60, and the probability of passing the math course is .70. The probability of passing both is .50. What is the probability of passing at least one?

Exercise 22. A study by the Ministry of tourism revealed that 50 percent of vacationers going to Lebanon visit Beirut Region, 40 percent visit the Montains, and 35 percent visit both.

- a. What is the probability a vacationer will visit at least one of these attractions?
- b. What is the probability .35 called?
- c. Are the events mutually exclusive? Explain.

Special Rule of Multiplication

- ▶ The rules of multiplication are applied when two or more events occur simultaneously
- ▶ The special rule of multiplication refers to events that are independent

INDEPENDENCE The occurrence of one event has no effect on the probability of the occurrence of another event.

SPECIAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A) P(B)$

A survey by the American Automobile Association (AAA) revealed 60% of its members made airline reservations last year. Two members are selected at random. What is the probability both made airline reservations last year?

$$P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$$

General Rule of Multiplication

- ▶ The general rule of multiplication refers to events that are not independent
- ▶ A conditional probability is the likelihood an event will happen, given that another event has already happened

CONDITIONAL PROBABILITY The probability of a particular event occurring, given that another event has occurred.

GENERAL RULE OF MULTIPLICATION

$$P(A \text{ and } B) = P(A)P(B|A)$$

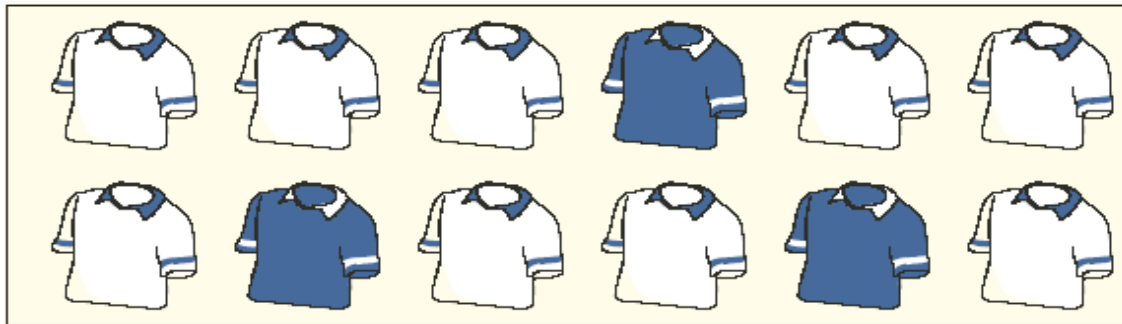
[5–6]

- ▶ The conditional probability is represented a $P(B|A)$ and is read, the probability of B given A

General Rule of Multiplication Example

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others are blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not return the shirts to the closet. What is the probability both shirts are white?

$$P(W_1 \text{ and } W_2) = P(W_1)P(W_2|W_1) = \left(\frac{9}{12}\right)\left(\frac{8}{11}\right) = .55$$



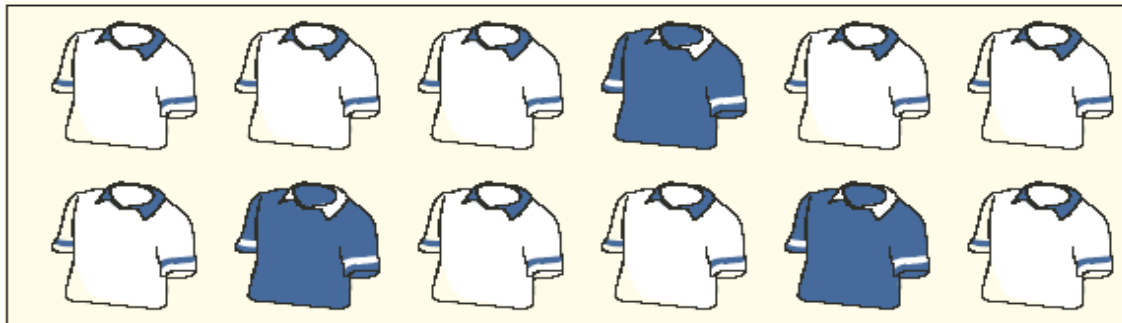
General Rule of Multiplication Example

We can extend the general rule of multiplication to more than two events. For three events A , B , and C , the formula is:

$$P(A \text{ and } B \text{ and } C) = P(A)P(B | A)P(C | A \text{ and } B)$$

In the case of the golf shirt example, the probability of selecting three white shirts without replacement is:

$$\begin{aligned} P(W1 \text{ \& } W2 \text{ \& } W3) &= P(W1)P(W2 | W1)P(W3 | W1 \text{ and } W2) \\ &= (9/12) (8/11) (7/10) = 0.38 \end{aligned}$$



Contingency Tables (1 / 2)

CONTINGENCY TABLE A table used to classify sample observations according to two or more identifiable categories or classes.

One hundred fifty adults were asked their gender and the number of Facebook accounts they used. The following table summarizes the results.

Gender									
Facebook Accounts		Men			Women			Total	
0			20			40			60
1			40			30			70
2 or more			10			10			20
	Total		70			80			150

Contingency Tables (2/2)

A sample of executives were surveyed about loyalty to their company. One of the questions was, “If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?” The responses of the 200 executives in the survey were cross-classified with their length of service with the company.

Loyalty	Length of Service				Total
	Less than 1 Year, B_1	1–5 Years, B_2	6–10 Years, B_3	More than 10 Years, B_4	
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

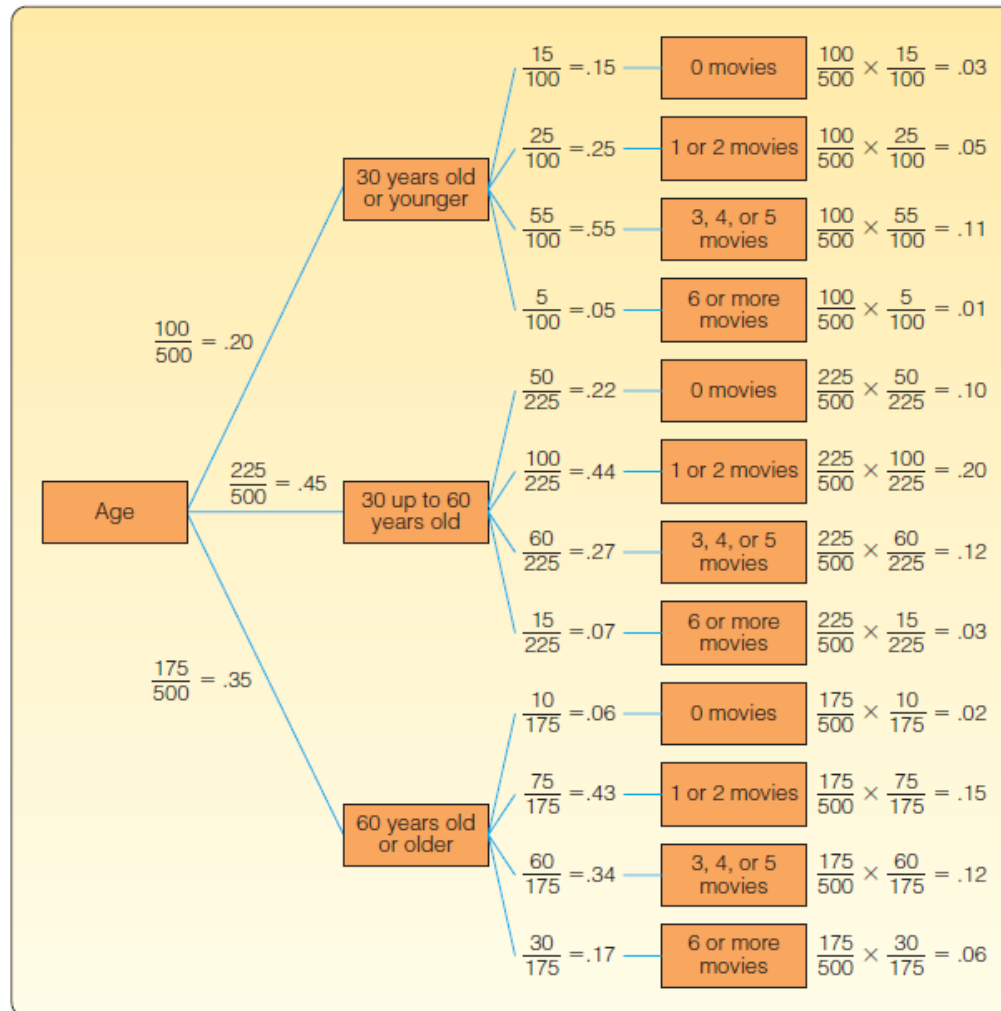
What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

Tree Diagrams

- ▶ A tree diagram is a visual that is helpful in organizing and calculating probabilities for problems with several stages
- ▶ Each stage of the problem is represented by a branch of the tree
- ▶ Label the branches with the probabilities

		Age									
		Less than 30			30 up to 60			60 or Older			
Movies per Month		B_1			B_2			B_3			Total
0	A_1		15			50			10		75
1 or 2	A_2		25			100			75		200
3, 4, or 5	A_3		55			60			60		175
6 or more	A_4		5			15			30		50
Total			100			225			175		500

Tree Diagram Example



Exercise (1 / 3)

Exercise 26: All Seasons Plumbing has two service trucks that frequently need repair. If the probability the first truck is available is .75, the probability the second truck is available is .50, and the probability that both trucks are available is .30, what is the probability neither truck is available?

Exercise 27: Refer to the following table.

- a. Determine $P(A1)$
- b. Determine $P(B1 | A2)$
- c. Determine $P(B2 \text{ \& } A3)$

Second Event	First Event			Total
	A_1	A_2	A_3	
B_1	2	1	3	6
B_2	1	2	1	4
Total	3	3	4	10

Exercise (2 / 3)

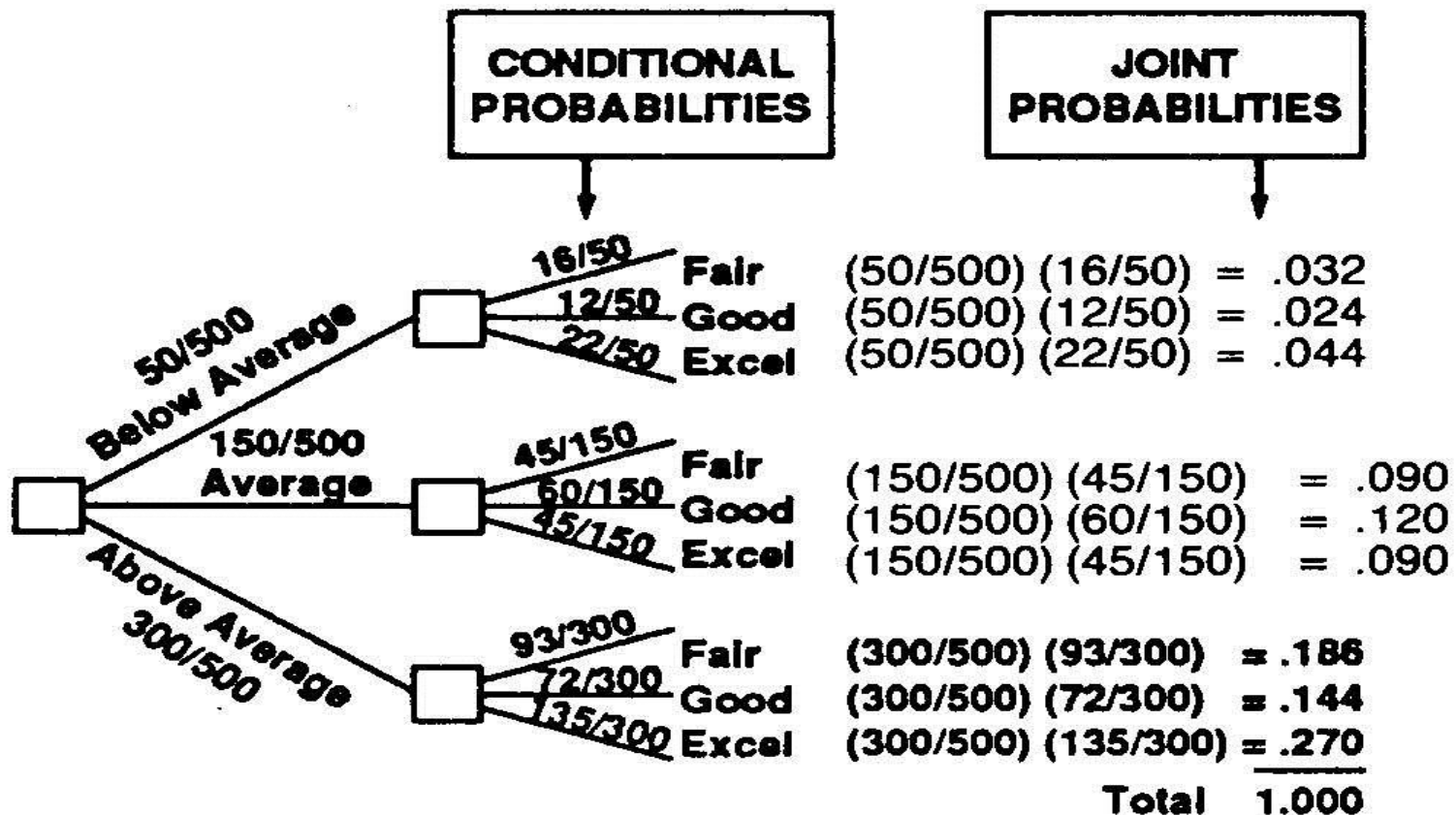
Exercise 29: Each salesperson at Puchett, Sheets, and Hogan Insurance Agency is rated either below average, average, or above average with respect to sales ability. Each salesperson is also rated with respect to his or her potential for advancement—either fair, good, or excellent. These traits for the 500 salespeople were cross-classified into the following table.

- What is this table called?
- What is the probability a salesperson selected at random will have **above average sales ability** and **excellent potential for advancement**?
- Construct a tree diagram showing all the probabilities, conditional probabilities, and joint probabilities.

Sales Ability	Potential for Advancement		
	Fair	Good	Excellent
Below average	16	12	22
Average	45	60	45
Above average	93	72	135

Exercise (3/3)

Exercise 29:



Bayes' Theorem

- ▶ Bayes' Theorem is a method of revising a probability, given that additional information is obtained
- ▶ For two mutually exclusive and collectively exhaustive events

BAYES' THEOREM
$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5-7]$$

PRIOR PROBABILITY The initial probability based on the present level of information.

POSTERIOR PROBABILITY A revised probability based on additional information.

Bayes' Theorem Example (1 / 2)

BAYES' THEOREM

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)}$$

[5-7]

Suppose 5% of the population of Umen have a disease and A_1 represents the part of the population that has the disease and A_2 represents those who do not. Let B denote a test result that shows the disease is present.

$P(A_1) = .05$ Individual has the disease

$P(A_2) = .95$ Individual does not have the disease

$P(B|A_1) = .90$ Test shows the individual has the disease and is correct

$P(B|A_2) = .15$ Test incorrectly shows the individual has the disease

Randomly select an individual and perform the test. The test results indicate the disease is present. What is the probability the test is correct? Use Bayes' theorem to solve.

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} = \frac{(.05)(.90)}{(.05)(.90) + (.95)(.15)} = \frac{.0450}{.1875} = .24$$

Bayes' Theorem Example (2/2)

Event, A_i	Prior Probability, $P(A_i)$	Conditional Probability, $P(B A_i)$	Joint Probability, $P(A_i \text{ and } B)$	Posterior Probability, $P(A_i B)$
Disease, A_1	.05	.90	.0450	$.0450/.1875 = .24$
No disease, A_2	.95	.15	.1425	$.1425/.1875 = .76$
			$P(B) = .1875$	1.00

- The probability of A_1 and A_2 are **prior probabilities** since their probabilities are assigned before any empirical data are obtained.
- B denotes the event “test shows the disease is present”. Assume the probability for a positive test result for someone with the disease is .90 and the probability of a positive test result is .15 for the individual who does not have the disease
- The probability $P(A_1|B)$ is called a **posterior probability** since it is a revised probability based on additional information. Using Bayes' theorem, we find the probability of someone who tests positive and actually has the disease. Therefore the probability that a person has the disease, given that he or she tested positive, is .24.

Exercise

Exercise 36: Dr. Stallter has been teaching basic statistics for many years. She knows that 80 percent of the students will complete the assigned problems. She has also determined that among those who do their assignments, 90 percent will pass the course. Among those students who do not do their homework, 60 percent will pass. Mike Fishbaugh took statistics last semester from Dr. Stallter and received a passing grade. What is the probability that he completed the assignments?

BAYES' THEOREM

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \quad [5-7]$$

$$\frac{0.80 \times 0.90}{(0.80 \times 0.90) + (0.20 \times 0.60)} = 0.8571$$

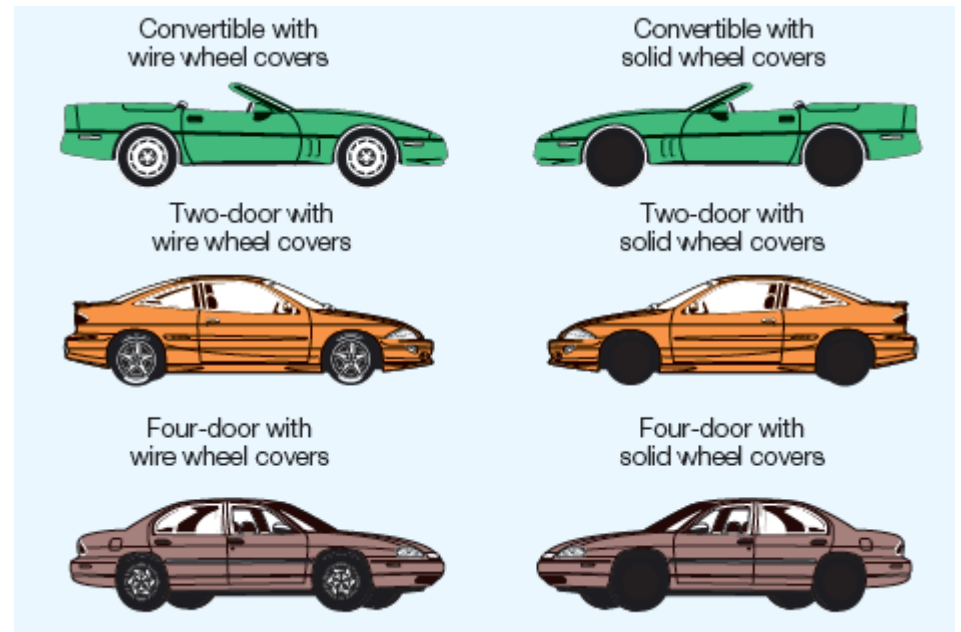
Multiplication Formula

- ▶ The multiplication formula states that if there are n ways of doing one thing, and m ways of doing another thing, then there are $m \times n$ ways of doing both

MULTIPLICATION FORMULA Total number of arrangements = $(m)(n)$ [5–8]

An automobile dealer wants to advertise that for \$29,999 you can buy a convertible, a 2-door, or a 4-door model with your choice of either wire wheel covers or solid wheel covers. How many different vehicles can the dealer offer?

Total possible = $(m)(n) = (3)(2) = 6$



The Permutation Formula (1/2)

- ▶ Another counting formula used to determine a total number of outcomes

PERMUTATION Any arrangement of r objects selected from a single group of n possible objects.

PERMUTATION FORMULA

$${}_nP_r = \frac{n!}{(n-r)!}$$

[5-9]

where:

n is the total number of objects.

r is the number of objects selected.

There are three electronic parts to be assembled, so $n=3$. Because all three are to be inserted into the plug-in component, $r=3$.

$${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 6$$

Label the parts A, B, and C ABC BAC CAB ACB BCA CBA

The Permutation Formula (2/2)

Betts Machine Shop Inc. has eight screw machines but only three spaces available in the production area for the machines. In how many different ways can the eight machines be arranged in the three spaces available?

There are eight possibilities for the first available space in the production area, seven for the second space (one has been used up), and six for the third space. Thus:

$$(8)(7)(6) = 336,$$

that is, there are a total of 336 different possible arrangements. This could also be found by using formula (5-9). If $n = 8$ machines, and $r = 3$ spaces available, the formula leads to

$${}_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{(8)(7)(6)\cancel{5!}}{\cancel{5!}} = 336$$

The Combination Formula (1 / 2)

- ▶ Another counting formula useful in determining the total number of outcomes
- ▶ A combination is an arrangement where the order of the objects selected is not important

COMBINATION FORMULA

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

[5-10]

where:

n is the total number of objects.

r is the number of objects selected.

The Grand 16 movie theater uses teams of three employees to work the concession stand each evening. There are seven employees available to work. How many different teams can be scheduled?

$${}_7C_3 = \frac{n!}{r!(n-r)!} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$$

The Combination Formula (2/2)

The marketing department has been given the assignment of designing color codes for the 42 different lines of compact disks sold by Goody Records. Three colors are to be used on each CD, but a combination of three colors used for one CD cannot be rearranged and used to identify a different CD. This means that if green, yellow, and violet were used to identify one line, then yellow, green, and violet (or any other combination of these three colors) cannot be used to identify another line. Would seven colors taken three at a time be adequate to color-code the 42 lines?

According to formula (5–10), there are 35 combinations, found by

$${}_7C_3 = \frac{n!}{r!(n-r)!} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$$

The seven colors taken three at a time (i.e., three colors to a line) would not be adequate to color-code the 42 different lines because they would provide only 35 combinations. Eight colors taken three at a time would give 56 different combinations. This would be more than adequate to color-code the 42 different lines.

Exercise

- ▶ **Exercise 44:** A representative of the Environmental Protection Agency (EPA) wants to select samples from 10 landfills. The director has 15 landfills from which she can collect samples. How many different samples are possible?
- ▶ **Exercise 46:** A company is creating three new divisions and seven managers are eligible to be appointed head of a division. How many different ways could the three new heads be appointed? Hint: Assume the division assignment makes a difference